ADI KAVI NANNAYA UNIVERSITY
SEMESTER END EXAMINATIONS
M. Sc. Applied Mathematics
IV-SEMESTER
M401: Functional Analysis
[W.E.F.2016 A.B.]
(Model Question Paper)

Time: 3 Hours **Max. Marks: 75**

Answer **ALL** questions. Each question carries 15 marks **Mark:** $5 \times 15 = 75$

1. a) Show that every finite dimensional subspace Y of a normed space X is complete. In particular, every finite dimensional normed space is complete.
   b) Prove that finite dimensional vector space is algebraically reflexive.

(OR)

2. a) If Y is a Banach space then, prove that (the set of all bounded linear operators from X into Y) B(X, Y) is a Banach space.
   b) If T be a linear operator, then, prove that the range, R(T) is a vector space.

3. State and prove Bessel Inequality.

(OR)

4. State and prove Minimizing vector Theorem.

5. State and prove Riesz- Representation Theorem

(OR)

6. a) Let H_1, H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operators and α any scalar. Then prove the following: i) $\langle T^* y, x\rangle = \langle y, Tx\rangle$ ii) $(\alpha T)^* = \overline{\alpha} T^*$ iii) $(T^*)^* = T$.
   b) Let the operators $U: H \to H$ and $V: H \to H$ be unitary and H is Hilbert space. Then, prove that abounded linear operator T on a complete Hilbert space H is unitary if and only if T is isometric and surjective.
7. State and prove Generalized Hahn-Banach Theorem.

(OR)

8. State and prove Open Mapping Theorem.

9. Answer any **THREE** questions of the following.
   a) State and prove Schwarz Inequality and Triangle Inequality.
   b) Define inner product space, orthogonality, Hilbert space and Banach space.
   c) State and prove Baire's Category Theorem in Complete metric space.
   d) Show that the dual space R^n is R^n.
   e) Show that an orthonormal set is linearly independent.

---------*---------

Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

ADI KAVI NANNAYA UNIVERSITY
SEMESTER END EXAMINATIONS
M.Sc. Applied Mathematics
IV-SEMESTER
A M402: OPERATIONS RESEARCH
[W.E.F.2016 A.B.]
(Model Question Paper)

Time: 3 Hours | Max. Marks: 75

Answer **ALL** questions. Each question carries 15 marks | Marks: $5 \times 15 = 75$

---

1. An insurance company has decided to modernize and refit one of its branch offices. Some of the existing office equipments will be disposed of but the remaining will be returned to the branch of completion of the renovation work. Tenders are invited from a number of selected contractors. The contractors will be responsible for all the activities in connection with the renovation work expecting the prior removal of the old equipment and its subsequent replacement.

The major elements of the project have been identified as follows along with their durations and immediately preceding elements.

Activity	Description	Duration (weeks)	Immediate Predecessors
A.	Design new premises	14	-
B.	Obtain tenders from the contractors	4	A
C.	Select the contractor	2	B
D.	Arrange details with selected contactor	1	C
E.	Decide which equipment to be used	2	A
F.	Arrange storage of Equipment	3	E
G.	Arrange disposal of other equipment	2	E
H.	Order new equipment	4	E
I.	Take delivery of new equipment	3	H, L
J.	Renovations take place	12	K
K.	Remove old equipment for storage Or disposal	4	D, F, G
L.	Cleaning after the contactor has finished	2	J
M.	Return old equipment for storage	2	H, L

a) Draw the network diagram showing the interrelations between the various activities of the project

b) Calculate the minimum time that the renovation can take from the design stage

(OR)

2. A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity Estimated Duration (weeks)

(i- j)	Optimistic	Most Likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

a) Draw the network diagram of activities in the project.

b) Find the expected duration and variance for each activity. What is the expected project length?

c) If the project due date is 19 weeks, what is the probability of not meeting the due date.

Given: Z :	0.50	0.67	1.00	1.33	2.00
Probability :	0.3085	0.2514	0.1587	0.0918	0.0228

3. Derive an EOQ Model with Constant Rate of Demand when shortages are not allowed, production is instantaneous.

(OR)

4. A commodity is to be supplied at a constant rate of 200 units per day. Supplies of any amount can be obtained at any required time, but each ordering costs Rs.50, cost of holding the commodity in inventory is Rs.2 per unit day while the delay in the supply of the item induces a penalty of Rs.10 per unit per day. Find the optimal policy (Q, t), where 't' is the reorder cycle period and 'Q' is the inventory after reorder. What would be the best policy, if the penalty cost becomes infinite?

5. Write about the performance measures for Model 1 {(M/M/1) : $(\infty/FCFS)$}

(OR)

6. Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 3 minutes.
   a. What is the probability that a person arriving at the booth will have to wait?
   b. The telephone department will install a second booth when convicted that an arrival would expect waiting for atleast 3 minutes for a phone call. By how much should the flow of arrivals increase in order to justify a second booth?
   c. What is the average length of the queue that forms from time to time?
   d. What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(OR)

7. Use the principle of optimality to find the maximum value of $z = b_1x_1 + b_2x_2 + b_3x_3 + \cdots \ldots . . b_nx_n$ when $x_1 + x_2 + x_3 + \cdots \ldots . +x_n = c$ and $x_1, x_2, x_3 \ldots . . x_n \geq 0, b_1 > 0, b_2 > 0 \ldots \ldots b_n > 0$

(OR)

8. Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing times (in hours) are given in the following table:

Job	: 1	2	3	4	5
Machine A	: 8	10	6	7	11
Machine B	: 5	6	2	3	4
Machine C	: 4	9	8	6	5

9. Answer any **THREE** questions of the following

   a. Explain Looping, Dangling and Dummy activity in network.
   b. Write the difference between PERT and CPM.
   c. Write a brief note on COST'S involved in inventory models.
   d. Derive expected number of customers waiting in the queue for Model 1 $\{(M/M/1):(\infty/FCFS)\}$
   e. Write about Bell-man's principle in Dynamic programming.

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc. Applied Mathematics, IV Semester

M403: Methods of Applied Mathematics

[W.E.F.2016 A.B.]

(Model Question paper)

Time: 3 Hours **Max.Marks:75**

Answer **ALL** questions. Each question carries 15 marks **Marks: 5x15=75**

1(i) Determine whether the vector {6,1,-6,2} is in the vector space generated by the vectors {1,1,-1,1}, {-1,0,1,1} and {1,-1,-1,0}.

(ii)Find the dimension of the vector space of the vector space generated by the vectors {1,1,0},{1,0,1} and {0,1,1}.

(OR)

2.(i)Prove that two characteristic vectors of a real symmetric matrix, corresponding to different characteristic numbers are orthogonal.

(ii)Show that all characteristic numbers of a real symmetric matrix are real.

3. Show that R(x, ζ ; λ)=k(x, ζ)+ $\lambda \int_{\zeta}^{x} k(x,z)R(z,\zeta;\lambda)dz$ is the solution of a non-homogeneous Volterra's integral equation of second kind $\phi(x) = f(x) + \lambda\int_0^x k(x,\zeta)\phi(\zeta)d\zeta$, where R(x, ζ ; λ) is the resolvent kernel of the equation.

(OR)

4. Solve the Volterra integral equation of second kind $\phi(x) = (1+x) - \int_0^x \phi(\zeta)d\zeta$

with $\phi_0(x) = 1$ by using the method of successive approximations.

5. State and prove Fredholm's Third theorem

(OR)

6. Using Fredholm determinants, find the resolvent kernels of the kernel k(x, ζ)=x e^{ζ} ; a=0, b=1

7.Determine the eigen values and eigen functions of homogeneous integral equation $\phi(x) = \lambda\int_0^1 (2x\zeta - 4x^2)\phi(\zeta)d\zeta$

(OR)

Dr. T. HYMAVATHI
Convener-P.G. B.O.S.
Applied Mathematics & Mathematics
Adikavi Nannaya University
Rajamahendravaram-533 296

8. Construct the Green's function for the boundary value problem $u''(x)+\mu^2 u=0$ with the conditions u(0)=u(1)=0.

9. Answer any **THREE** questions of the following :

a) State and prove Schwarz Inequality.

b) Show that the function $\phi(x)=(1+x^2)^{-\frac{3}{2}}$ is a solution of the Volterra integral equation

$$\phi(x)=\frac{1}{1+x^2}-\int_0^x \frac{\zeta}{1+x^2}\phi(\zeta)\,d\zeta$$

c) Find the solution of the integral equation $\phi(x)=1+\int_0^x (\zeta-x)\phi(\zeta)d\zeta$

d) Determine $D(\lambda)$ and $D(x,\zeta;\lambda)$ for the kernel $k(x,\zeta)=1$ with the limits 0 and 1.

e) Write the properties of Green's function.

*****

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc APPLIED MATHEMATICS

IV SEMESTER

AM(404) INTEGRAL TRANSFORMS

(w.e.f.2016 AB)

MODEL QUESTION PAPER

Time : 3Hours | Max.Marks: 75

Marks : 5 X 15 = 75

Answer ***ALL*** Questions and ***Each Question*** Carries **15** Marks

1. Prove the following Hypothesis:
If F(t) is continuous for all t≥0 and be of exponential order a as t $\to \infty$ and if F^1(t) is of class A, then the Laplace transformation of the derivative F^1(t) exist when p> a and

L[F'(t)] = p L[F(t)] – F(0)

(OR)

2.Find the Inverse Laplace Transformation of the following functions

(a) $\frac{2P+1}{(P+2)^2(P-1)^2}$ (b) $\frac{e^{-4P}}{(P-3)^4}$

3.Find the Fourier transform of F(t) defined by

F(x) = 1 for IxI < a

= 0 for IxI > a and hence evaluate the following

(a) $\int_{-\infty}^{\infty} \frac{SinPa\ CosPx}{P}\, dP$ (b) $\int_0^{\infty} \frac{SinP}{P}\, dP$

(OR)

4.Find Fourier Sine and Cosine transform of e^{-x} and using the inversion formulae recover the original functions, in both the cases

5.(a) Find the Fourier Sine and Cosine Transforms of f(x) = x

(b) When f(x) = sin mx, where m is a positive integer show that $\tilde{f}_s$(p) = 0 if p$\neq m$ and show that $\tilde{f}_s$(p) $=\frac{\pi}{2}$ if p = m

(OR)

6.Find the finite sine transform of f(x), if

(a) f(x) = cosKx (b) f(x) = x^3 and (c) f(x) = e^{cx}

7.Find Hankel Transform of the following function f(x) = 1 for 0< $x < a$, n=0

= 0 for x > $a,\ n = 0$

(OR)

8. Derive The Hankel Transform Expression for derivatives of a function

9.Answer any three form the following

(a) Find the L{ e^{3t} (3 Sin2t – 2 Cos 2t) }

(b) Find Inverse Laplace Transformation of $\frac{3}{(P-4)(P+5)}$ using Convolution Theorem

(c) Define Fourier Transform and Complex Fourier Transform

(d) Define Finite Fourier Sine Transformation

(e) State and Prove the Linearity property for Hankel Transformation

ADIKAVI NANNAYA UNIVERSITY

SEMESTER END EXAMINATIONS

M.Sc APPLIED MATHEMATICS

IV SEMESTER

AM(405) GRAPH THEORY

(w.e.f.2016 AB)

MODEL QUESTION PAPER

Time : 3Hours

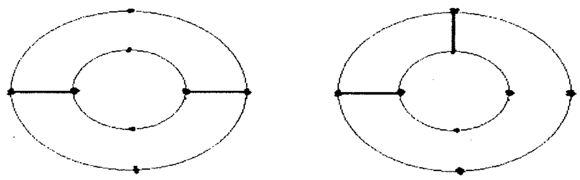
Max.Marks: 75
Marks : 5 X 15 = 75

Answer ***ALL*** Questions and ***Each Question*** Carries ***15*** Marks

1. (a) Prove that "The number of vertices of odd degree in a graph is even"
   (b) Explain the Konig's berg Bride problem

(OR)

2. Test whether the following graphs are Isomarphic or not

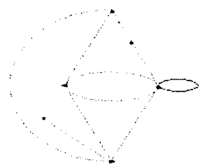


(A) (B)

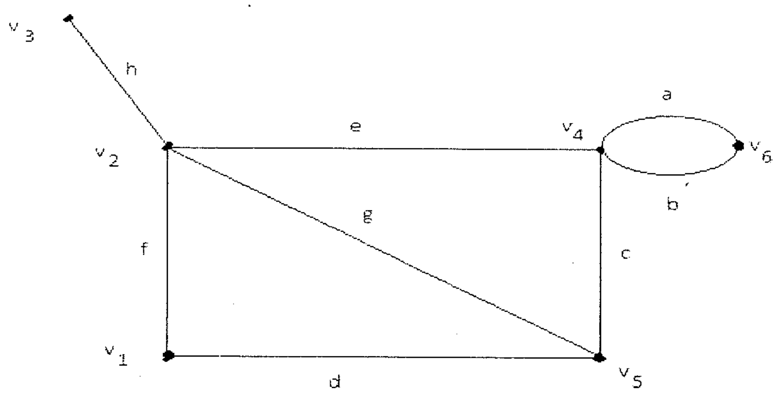
3. Prove that the complete graph of five vertices is non-planar

(OR)

4. Obtain the Dual of the following graph



5. Let A and B be the respective circuit matrix and the incidence matrix (of a self loop free graph) whose columns are arranged using the same order of edges. Then every row of B is orthogonal to every row A .i.e. $A.B^T = B.A^T$ = O(Mod 2)
   Verify the result for the following graph



(OR)

(6) If A(G) is an incidence matrix of a connected graph G with n vertices, then show that the rank of A(G) is n-1

(7) Show that the vertices of every planar graph can be properly colored with five colors

(OR)

(8) State and prove Max-flow-min –cut theorem

(9) Answer any THREE of the following

(a)) Define Incidence Matrix and Path Matrix.

(b) Define Tree, Binary Tree with examples

(C) Show that Kuratowski second graph is non planar

(d) Show that every tree with two or more vertices is two chromatic number

(e) Obtain the adjacency matrix of the following graph